

Modelling of Curved Creased Origami Structures

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Abstract

This paper aims to develop an analytical model to predict the mechanical behaviors of origami structures with curved creases. A basic curved creased unit cell is illustrated in Fig. 1(a). The crease line of the unit cell in its unfolded plane state is defined by a single period of an arbitrary periodic smooth function $y = f(x)$ with period T . The longitudinal edge length is a . In the unfolded plane state, the angle between the tangential of the crease line at location $x = x_a$ and the y axis, $\gamma(x_a)$, and the differential length of the crease line at location $x = x_a$, $b(x_a)$, are found as

$$\gamma(x_a) = \tan^{-1} \frac{1}{f'(x_a)} \quad (1)$$

$$b(x_a) = \frac{dx}{\sin \gamma(x_a)} = \sqrt{1 + f'(x_a)^2} dx \quad (2)$$

where dx is the differential length defined along the x axis.

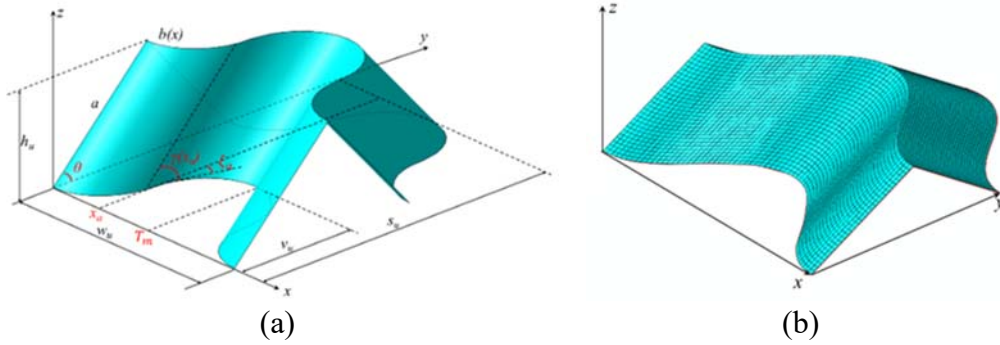


Figure 1: (a) The geometry of the curved-creased unit cell. (b) The FE model of the unit cell.

During the folding of the unit cell, the angle between the tangent of the folded crease line at location $x = x_a$ and the y axis, denoted by $\xi_a \in [0, \gamma(x_a)]$, is employed as the parameter to describe the folding motion. The outer dimensions of the unit cell are obtained as

$$w_u = \int_0^T \sqrt{1 + f'(x)^2 - \frac{f'(x)^2 \cos^2 \xi_a}{C(x_a)^2}} dx \quad (3)$$

$$s_u = \frac{2aC(x_a)}{\cos \xi_a} + v_u \quad (4)$$

$$v_u = \int_0^{T_m} \frac{f'(x) \cos \xi_a}{C(x_a)} dx = \frac{\cos \xi_a}{C(x_a)} (f(T_m) - f(0)) \quad (5)$$

$$h_u = aC(x_a) \sqrt{\frac{1}{f'(x_a)^2} - \tan^2 \xi_a} \quad (6)$$

where $C(x)$ is the cosine function of $\gamma(x)$, i.e.

$$C(x) = \frac{f'(x)}{\sqrt{1 + f'(x)^2}} \quad (7)$$

The in-plane and two out-of-plane Poisson's ratios of the unit cell are then derived as $\nu_{sw} = -\varepsilon_s/\varepsilon_w$, $\nu_{wh} = -\varepsilon_w/\varepsilon_h$ and $\nu_{sh} = -\varepsilon_s/\varepsilon_h$, respectively, where ε_w , ε_s and ε_h are the infinitesimal strains in the W , S , and H directions, respectively. Using the minimum total potential energy principle, the total potential energy E of the unit cell can be expressed as

$$E = U - \int_{\xi_{10}}^{\xi_1} f_x \frac{dw}{d\xi'_a} d\xi'_a - \int_{\xi_{10}}^{\xi_1} f_y \frac{ds}{d\xi'_a} d\xi'_a - \int_{\xi_{10}}^{\xi_1} f_z \frac{dh}{d\xi'_a} d\xi'_a \quad (8)$$

where f_w , f_s and f_h are the external forces applied in the W , S and H directions, respectively. The external forces at equilibrium state are then obtained by enforcing the condition $\delta E/\delta \xi_a = 0$.

To validate the analytical model, a finite element (FE) model of the unit cell is developed in the FE software ABAQUS, see Fig. 1(b). The simulation and analytical results are in good agreement, as shown in Fig. 2, which implies the validity of the analytical model. The analytical model developed in this paper can be readily extended to modelling periodic origami tessellations and stacked bulk material with multiple different layers.

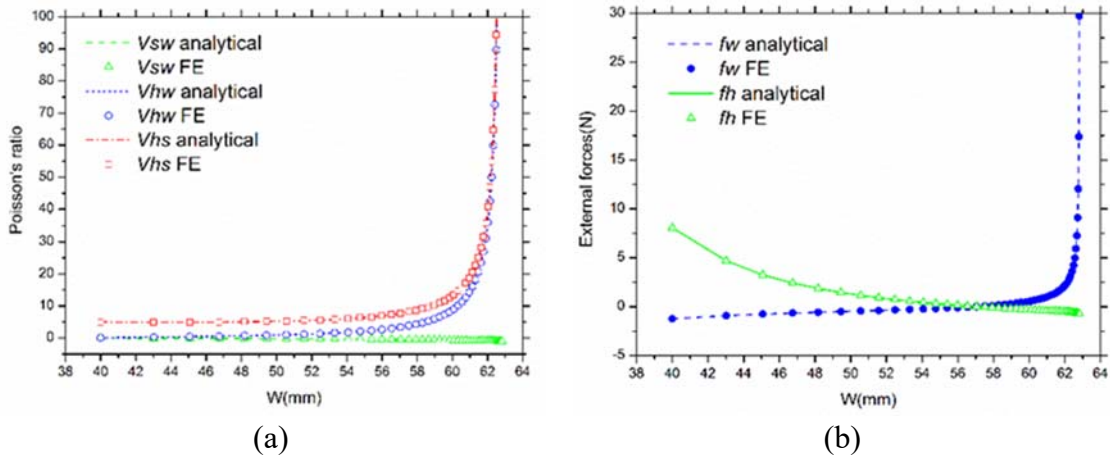


Figure 2: Comparison of the FE and analytical results: (a) Poisson's ratios; (b) External forces.